Three Topocentric Range Measurements to Pioneer 10 Near Jupiter Encounter and a Preliminary Estimate of an Earth Barycenter to Jupiter Barycenter Distance

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By using ground digitally controlled oscillator (DCO) apparatus installed at Goldstone (DSS 14) and Ballima, Australia (DSS 43), ramped carrier frequency doppler data were received from Pioneer 10 just prior to and shortly after Jupiter encounter. The analysis of these DCO doppler data resulted in three independent topocentric range measurements. These range measurements were individually accurate to at least ±5 km. The observed accuracy was 500 m but because of suspected systematic errors which were masked by the orbit adjustment procedure the actual accuracy was probably larger than 500 m. From the data residuals based on local orbital adjustments, the DCO data were no different from conventional doppler. The data error was on the order of 2 mHz. A longer solution was also attempted, whereby a Jupiter barycenter to Earth barycenter distance was found. The difference between the estimated barycenter-to-barycenter distance and that from the reference planetary ephemeris DE84 was −107 km. This difference was not significant because DE84 has a one standard deviation error of 250 km.

I. Introduction

Three passes of Pioneer 10 data near Jupiter using the Deep Space Network's digitally controlled oscillator (DCO) devices at Goldstone (DSS 14) and Ballima, Australia (DSS 43) were analyzed. Two of the passes of data were taken about 10 days before encounter and the last pass about a week after encounter. Each of the passes possessed a 100-Hz/s S-band sawtooth carrier frequency pattern, which lasted about 20 min. The data were reduced to three topocentric spacecraft range estimates with an estimated accuracy of 5 km in one-way range. The random noise observed on all of these passes was about 1.5 mHz. A larger data sample comprising of a continuous set beginning on November 23, 1973 and ending on December 13, 1973 was also analyzed. This larger set contained the three aforementioned DCO data; this entire span of data was used to infer an Earth–moon barycentric to Jupiter + moon barycentric distance at Jupiter encounter on December 4, 1973.

This determination was compared with the barycenter-to-barycenter distance as tabulated by JPL Development Ephemeris 84 (DE84), an ephemeris which was based on previous optical observations. This ephemeris placed an uncertainty of ±250 km in the barycenter-to-barycenter distance on December 4, 1973. Our orbit reduction gave a shift of −107 km from DE84 in the barycenter-to-barycenter distance. This shift is within the acknowledged uncertainty of ±250 km in this coordinate of DE84.

II. Analysis and Measurement Errors

The received observable ϕ is the difference between the accumulated (integrated) cycles at time t of received signal and a locally accumulated reference. Assume that at time t_1, $C(t_1)$ cycles were transmitted to the spacecraft. At a later time t_3, $C(t_1)$ is received again at the ground. The ground reference cycle is $C(t_3)$. Assume further that there were no cycles "lost" between transmission and reception due to troposphere, ionosphere, etc. The observable $\phi(t_3)$ is then:

$$\phi(t_3) = C(t_3) - C(t_1) + \phi_0 \tag{1}$$

where ϕ_0 is an arbitrary number of cycles or phase and

$$C(t) = \int_{t_0}^{t} \omega(\tau)\, d\tau \tag{2}$$

$$C(t) = \int_{t_0}^{t} [\omega_0 + \dot{\omega}(\tau - t_0)]\, d\tau$$

$$= \omega_0 (t - t_0) + \frac{\dot{\omega}}{2} (t - t_0)^2$$

where

ω = phase rate

τ = station time

t_0 = ramp initiation time

ω_0 = some reference frequency

$\dot{\omega}$ = frequency ramp

Thus, at reception time t_3, the observable is

$$\begin{aligned} \phi(t_3) &= \omega_0 (t_3 - t_1) + \frac{\dot{\omega}}{2} [(t_3 - t_0)^2 \\ &\quad - (t_1 - t_0)^2] + \phi_0 \\ &= \omega_0 (t_3 - t_1) + \frac{\dot{\omega}}{2} [(t_3 - t_1)^2 \\ &\quad + 2(t_3 - t_1)(t_1 - t_0)] + \phi_0 \end{aligned} \tag{3}$$

We recognize $(t_3 - t_1)$ to be the round-trip time of the signal.

Let

c = speed of light

$R = c(t_3 - t_1)$ = two-way range

$$\phi(t_3) = \frac{\omega_0 R}{c} + \frac{\dot{\omega}}{2c^2} R^2 + \frac{\dot{\omega}}{c} R (t_1 - t_0) + \phi_0 \tag{4}$$

At a later or subsequent reception time t'_3, the observable is

$$\phi(t'_3) = \frac{\omega_0 R'}{c} + \frac{\dot{\omega}}{2c^2} R'^2 + \frac{\dot{\omega}}{c} R' (t'_1 - t_0) + \phi_0 \tag{5}$$

To eliminate the unknown arbitrary constant ϕ_0, the phase counts at t'_3 and t_3 are differenced:

$$\begin{aligned} \Delta\phi(T) &= \phi(t'_3) - \phi(t_3) \\ &= \frac{\omega_0 (R' - R)}{c} + \frac{\dot{\omega}}{c} [R' (t'_1 - t_0) \\ &\quad - R(t_1 - t_0)] + \frac{\dot{\omega}}{2c^2} (R'^2 - R^2) \end{aligned} \tag{6}$$

The differenced cycle or phase count $\Delta\phi$ can be associated with any time between t_3 and t'_3. Customarily, the observable is time tagged half way between t'_3 and t_3:

$$T = t_3 + \frac{1}{2} (t'_3 - t_3) \tag{7}$$

Notice from Eq. (6) that conventional differential phase ($\dot{\omega} = 0$) will tell us the range change from time t_3 to t'_3. With the DCO data ($\dot{\omega} \neq 0$), we can also determine range. To see this, assume a static case where the transmitter, spacecraft, and receiver are stationary (such as might be the situation for a geostationary satellite). Then

$$R' = R$$

and

$$t'_1 - t_1 = t'_3 - t_3 = \tau$$

where τ is the doppler averaging time. Differential phase

$$\Delta\phi(T) = \frac{\dot{\omega}}{c} R\tau \tag{8}$$

Whereas conventional data with a zero frequency rate have zero differential phase, the DCO or ramp frequency data (differential phase between two successive data readouts) have values which are proportional to the round-trip distances.

From Eq. (8), we find the expression relating the phase errors ($\delta\phi$) and the resulting range error (ΔR) to be:

$$\Delta R = \frac{c}{T}\frac{\delta\phi}{\dot{\omega}} \tag{9}$$

where

ΔR = two-way range error, km

c = speed of light, 2.99×10^5 km/s

$\dot{\omega}$ = S-band ramp rate, Hz/s

T = ramp averaging time, s

$\delta\phi$ = phase error, cycles

Additionally, an error in the ramp initiation time appears directly in a round-trip range error. The expression is

$$\Delta R = c\Delta t \tag{10}$$

where Δt is the ramp initiation time error in seconds. A discussion of each individual error follows.

A. Phase Error (Short Term) or "Jitter"

From Ref. 1 a test was conducted whereby phase jitter could be measured. The phase noise has a standard deviation scatter of 0.02 cycles. For a ramp rate of 100 Hz/s and an averaging time of 600 s, the two-way range error from this is

$$\Delta R = \frac{3 \times 10^5 \times 0.02}{100 \times 600} = 0.1 \text{ km (100 m)}$$

B. Phase Drift

Reference 2 gives a value of the stability for the rubidium standard as 5 parts/10^{12}, which gives an S-band frequency error of 11.5×10^{-3} Hz, or about 7 cycles in 10 min. The range error for a ramp rate of 100 Hz/s for 10 min is

$$\Delta R = \frac{3 \times 10^5 \times 7}{6 \times 10^4} = 35 \text{ km}$$

Reference 2 also points out that the rubidium standard has been improved by a factor of 10 so that the range error is

$$\Delta R = 3.5 \text{ km}$$

C. Tropospheric Effects

The number of cycles added is a function of elevation angle of the spacecraft. If we assume that the elevation angle is above 20 deg, then the addition or subtraction of 10 refractivity (N) units (refractivity unit = (index of refraction minus 1) $\times 10^6$) will add or subtract about a cycle during one hour. Ten N units correspond to a 3% error in the troposphere. The error due to a 3% change over an hour of the troposphere causes a 1/6-cycle error in 10 min. For a ramp averaging time of 10 min and a rate of 100 Hz/s:

$$\Delta R = \frac{3 \times 10^5}{600 \times 100 \times 6} = 0.83 \text{ km}$$

D. Ramp Initiation Time Error

References 1 and 3 indicate a 5-μs uncertainty in the initiation of the ramp. This uncertainty is the time quantization limit of the equipment. Translated into range, we have:

$$\Delta R = 3 \times 10^5 \times 5 \times 10^{-6} = 1.5 \text{ km}$$

Table 1 summarizes the various errors and the corresponding two-way range measurement error. The limiting or the largest error is caused by the instability or drift of the frequency standard. This error is about 1.5 km (one way), and is larger than errors due to the troposphere or to ramp-on-time errors.

III. Data Equation

The track synthesizer frequency (TSF) is controlled by the digitally controlled oscillator (DCO). The DCO frequency is related to the TSF by:

DCO frequency = 3 TSF − 20 MHz

DCO frequency rate = 3 TSF rate

TSF is typically around 22 MHz. For conventional doppler the TSF is multiplied by 96 so that the transmission to the spacecraft is near 2200 MHz (S-band frequency). The spacecraft multiplies the received frequency by

240/221 and transmits it to Earth. Thus the received frequency, ω (received) at the station, with all relative doppler motion removed is

$$\omega\,(\text{received}) = 96 \times \frac{240}{221} \times \text{TSF}$$

For the new swept DCO carrier frequency, the initial DCO frequency is ω_0 for initiation of ramp-on time t_0. The DCO frequency ω at any later time t is $\omega = \omega_0 + \dot{\omega}\,(t - t_0)$.

A 20-MHz frequency is added to this signal and the result multiplied by 32. This signal is at S-band and is transmitted to the spacecraft. As in conventional doppler, the up-signal is received by the spacecraft and multiplied by 240/221 and transmitted back to Earth. We have this situation:

$$\omega_0 = (3 \times \text{TSF} - 20 \times 10^6)\ \text{Hz} \approx 46\ \text{MHz initially.} \tag{11}$$

At any later time t, the frequency is

$$\omega = \omega_0 + \dot{\omega}\,(t - t_0)$$

The transmitted frequency ω_T is

$$\omega_T = 32 \times (\omega + 20 \times 10^6) \tag{12}$$

The received signal is at a frequency ω (received):

$$\begin{aligned}
\omega\,(\text{received}) &= \frac{240}{221}\,\omega_T \\
&= 32 \times \frac{240}{221}\,(\omega + 20 \times 10^6) \\
&= 32 \times \frac{240}{221}\,[\omega_0 + \dot{\omega}\,(t - t_0) + 20 \times 10^6] \\
&= 32 \times \frac{240}{221}\,[3 \times \text{TSF} - 20 \times 10^6 \\
&\qquad + \dot{\omega}\,(t - t_0) + 20 \times 10^6] \\
&= 96 \times \frac{240}{221} \times \text{TSF} + 32 \times \frac{240}{221}\,\dot{\omega}\,(t - t_0)
\end{aligned} \tag{13}$$

The first term is the received S-band frequency for conventional doppler. The second term is the additional frequency introduced by the DCO device.

This analysis forms the basis for our data reduction scheme.

IV. Data Analysis

We analyzed a data span which began on November 23, 1973, 03:34:32 GMT and ended on December 13, 1973, 11:33:32 GMT. This data set encompassed the Jupiter encounter by Pioneer 10 which occurred on December 4, 1973, 02 h GMT. Also included in these data were three sets of DCO carrier ramps. These ramps occurred on November 23 and 26, 1973 and December 11, 1973. Table 2 tabulates the ramp pattern for those dates.

On each of those dates, a sawtooth pattern is impressed upon the S-band carrier by the DCO device. This pattern consists of an upswing of the carrier of 100 Hz/s for 5 min, followed by a downswing of -100 Hz/s for 10 min and another upswing of 100 Hz/s for 5 min. The final carrier frequency at the end of this saw-tooth is at the same frequency as at the beginning of the sawtooth. There is a known ramp initiation time delay of 1.000040 s which our data reduction scheme automatically takes into account. The 5-μs random on-time error discussed in Section II-D is not compensated and appears as a random range measurement error.

The ramp information presented in Table 2 was passed to an orbit determination program POEAS. This program automatically least square adjusted the orbit to the data whose representation was given by Eq. (13) (with the addition of appropriate extra terms for doppler motion). The orbit adjustments were done separately for the three passes containing the DCO data (November 24, 26, and December 11, 1973).

The data for each local orbit solution were comprised of normal unramped doppler for about an hour before the onset of the ramped (DCO) doppler data. The station locations for DSS 14 and DSS 43 are given in Table 3. Since the uncertainties of these locations amount to less than 5 m, we assumed these locations to be fixed and no adjustments to these parameters were made. Each of these orbit determinations resulted in a topocentric range measurement. The individual range measurements are tabulated in Table 4. The indicated uncertainties for these measurements were estimated by POEAS to be ± 5 km. This figure is very different from the 100-m error mentioned in Section II-A. The explanation is that the assumed doppler data accuracy figure given to POEAS is about 1 Hz for 1-s doppler averaging time or about 50 times larger than the ground test noise measurement. The less accurate figure given to the program was used as an attempt to account for systematic effects such as those mentioned in Sections II-A, -B, -C, -D. The orbit determination process removes systematic effects by interpreting these errors as errors in the orbit and adjusts it accordingly. The actual observed data noise after the orbit adjustments was in fact about 0.1 Hz, or 10 times smaller than given to the program. Figure 1 is a plot of doppler residuals for November 26, 1973, from DSS 14. The residuals have a one standard deviation ($1\text{-}\sigma$) spread of about 2 mHz. The ramped data on the transmission, which was not delivered to us by the project, are shown as missing in the figure. The ramped return signal, which was delayed by 1½-h round-trip time is marked off on Fig. 1. There was no difference between the conventional data and DCO data as seen from Fig. 1.

Figure 2 shows the number of S-band cycles that are gained or lost over the 2-h span of data from DSS 14. On the whole, there is about a cycle or 13-cm error during this entire period.

A longer span of data was also analyzed. This set of data began on November 23, 1973 and extended past encounter for 7 more days, ending on December 11, 1973. The purpose here was to use the 3-ramped DCO data as range data in combination with standard doppler data in order to determine a Earth–moon barycenter to Jupiter–Galilean moon barycenter distance. The standard unramped doppler data would yield an orbit relative to the Jovian center, and the DCO doppler data would determine the topocentric distance to the spacecraft. A combination of the two data types would reveal errors in the JPL planetary ephemeris DE84. The determination of the spacecraft orbit relative to Jupiter was, however, complicated by the perturbative effects caused by the four large moons, and the nonspherical gravity field of Jupiter. In order to separate these important effects from a misplacement of the position of Jupiter, it was necessary not only to adjust the spacecraft orbit, but also to adjust simultaneously the masses of the four moons, mass of Jupiter, and the heliocentric coordinates of Earth and Jupiter. This was attempted and the results of such an adjustment leading to an estimation of the barycenter-to-barycenter distance are summarized in Table 5. There is a slight displacement of Jupiter of about -107 km at encounter from DE84. This displacement is not significant since the DE84 coordinates have a one standard ($1\text{-}\sigma$) deviation of about 250 km at that time. Further, POEAS was extensively modified to allow for the additional perturbative effects of the four moons and a code error was incurred which prevented a Jovian mass adjustment. Because of this, a good orbit solution to the level of 10 mHz was not possible. Just prior to encounter, the orbit was warped to produce about 0.5-Hz residual in doppler. Throughout the orbit convergence procedure, however, the barycenter-to-barycenter distance estimation never deviated more than 300 km from DE84, indicating a stable solution in this respect.

V. Summary of Results

By using the DCO doppler data, we were able to obtain three range estimates to Pioneer 10. These range measurements were, individually at least, accurate to ± 5 km. The observed accuracy was 500 m but, because of suspected systematic errors which were masked by the orbit adjustment procedure, the actual accuracy was probably larger than 500 m. From the data residuals based on local orbital adjustments, the DCO data were no different from conventional doppler. The data error was on the order of 2 mHz. A longer solution was also attempted, whereby a Jupiter barycenter to Earth barycenter distance was found. The difference between the estimated barycenter-to-barycenter distance and that from the reference planetary ephemeris DE84 was -107 km. This difference was not significant because DE84 had a $1\text{-}\sigma$ error of 250 km.

References

1. Spradlin, G., *DSS 43 Ramp Test of 21 September 1973*, IOM 421-PF-A506, Oct. 2, 1973 (JPL internal document).

2. Cain, D. L., *Hydrogen Masers for Pioneer 10*, IOM 391.4-561, Sept. 12, 1973 (JPL internal document).

3. Berman, A., *Analyses of DCO Ramp Data*, IOM 431-G-73-57, July 11, 1973 (JPL internal document).

Table 1. Two-way range errors due to various sources for ramp rate 100 Hz/s (S-band) and 10-min averaging

Source	Error	Range error (two way), km
Phase noise (jitter)	0.02 cycles	0.1
Phase drift	4.2 cycles/h	3.5
3% Tropospheric error (EL > 20°)	1 cycle/h	0.83
Time initiation	5×10^{-6} s	1.5

Table 2. Ramp frequency pattern near Jupiter encounter

Date	Day of year	DSS	Ramp	DCO ramp rate, Hz/s	Ramp-on time,[a] GMT
11/24/73	328	43	1	3.125	04:00:00
			2	−3.125	04:05:00
			3	3.125	04:15:00
			4	0.0	04:20:00
11/26/73	330	14	1	3.123	00:05:00
			2	−3.123	00:10:00
			3	3.123	00:20:00
			4	0.0	00:25:00
12/11/73	345	43	1	3.125	04:05:00
			2	−3.125	04:10:00
			3	3.125	04:20:00
			4	0.0	04:25:00

[a]Conventional command time. Actual start time will be 1.000040 seconds later.

Table 3. Station locations referred to 1903.0 pole

DSS	Longitude, deg	Distance from rotation axis, km	Distance from equatorial plane, km
14	243.1104806	5203.9952949	3677.052
43	148.981274	5205.2472152	−3674.788

Table 4. Estimated topocentric distance to Pioneer 10[a]

DSS	Date	UTC time, GMT	Two-way light time,[b] UTC second	One-way[c] range, km
43	11/24/73	05:28:26	5316.70302147	796,953,845.29
14	11/26/73	01:43:50	5351.76510073	802,209,520.47
43	12/11/73	05:48:22	5600.36541890	839,473,774.94

[a]Measurement error of ±5 km (one way).
[b]Round-trip light time error is $\pm 0.3 \times 10^{-4}$ s.
[c]Speed of light is defined as 299792.5 km/s.

Table 5. Estimated barycenter-to-barycenter distance on December 4, 1973 0 hour GMT

Reference ephemeris distance (DE84), km	825,852,685.77
Correction to reference ephemeris, km	−107.89[a]
Corrected ephemeris distance, km	825,852,577.87[a]

[a]Measurement error of ±5 km.

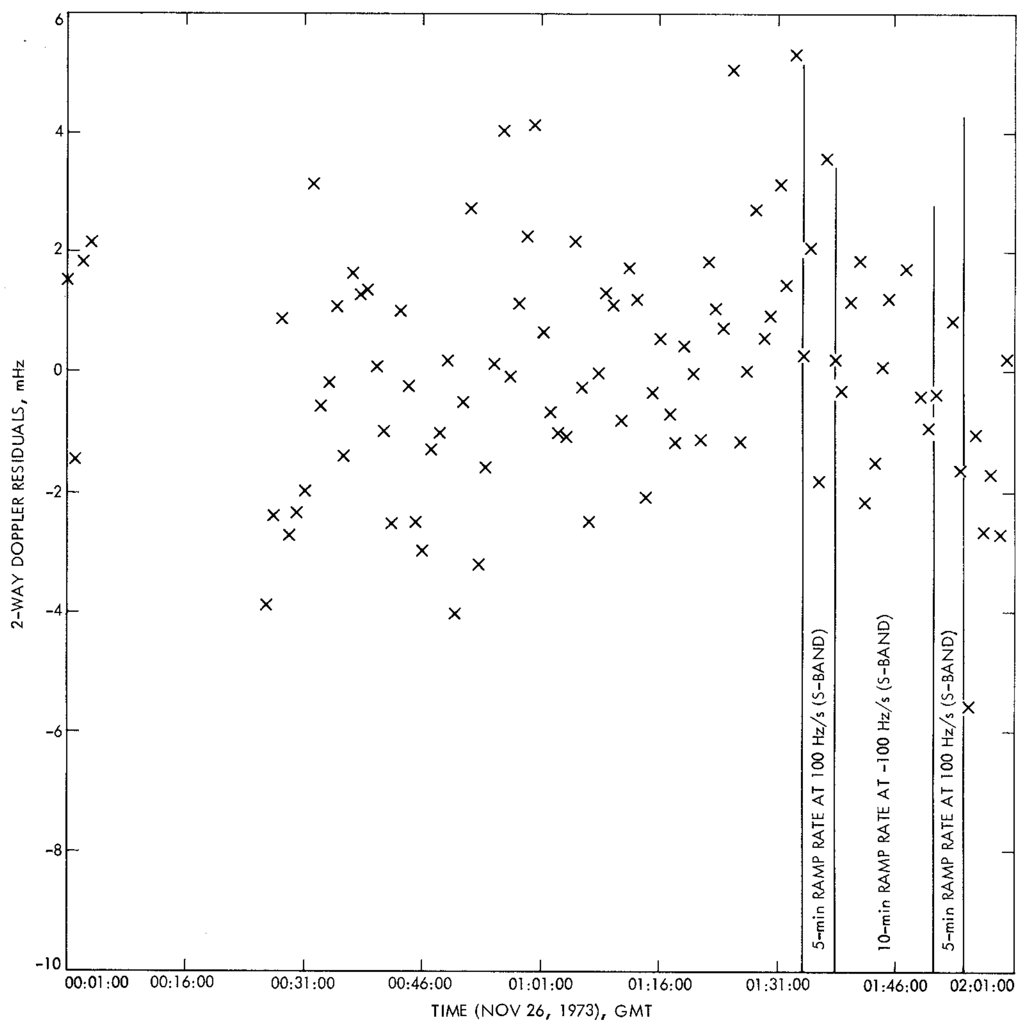

Fig. 1. Pioneer 10 doppler residuals from DSS 14 on November 26, 1973

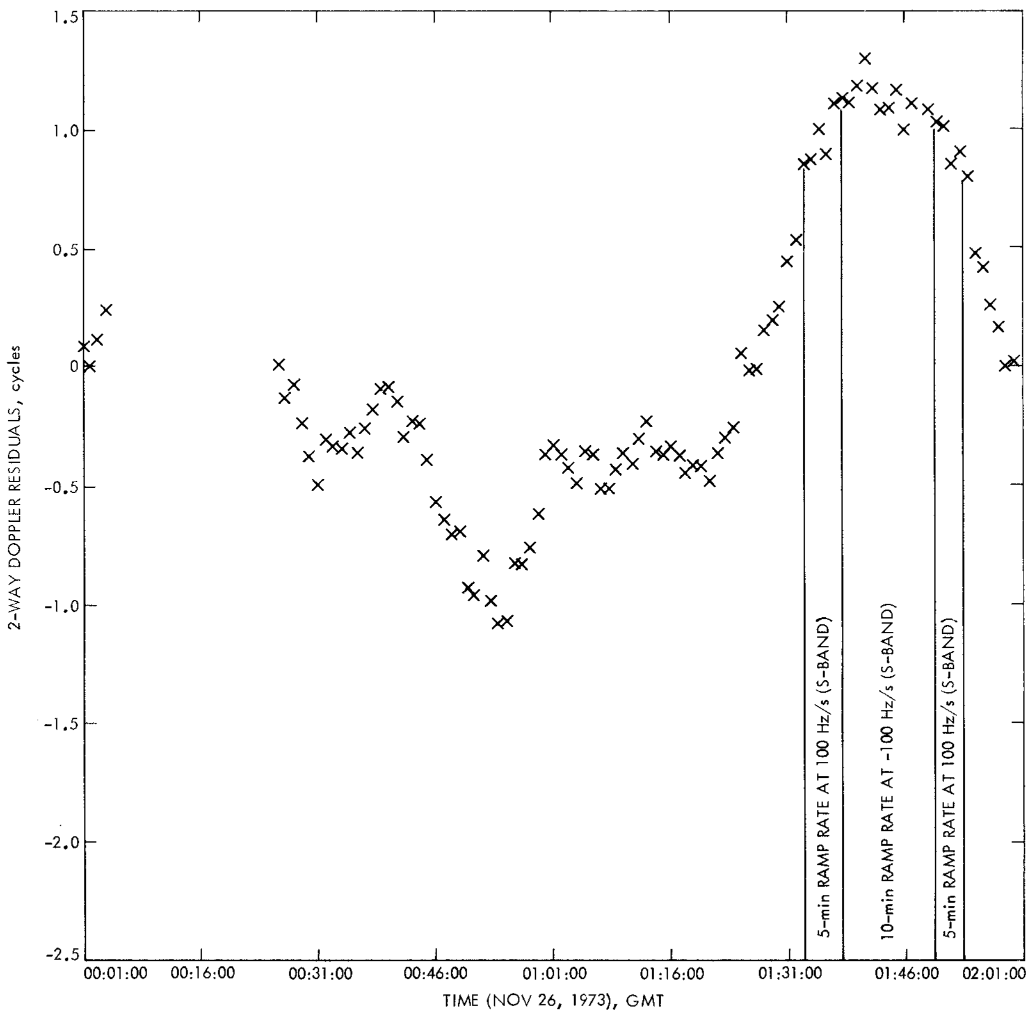

Fig. 2. Pioneer 10 S-band phase residuals from DSS 14 on November 26, 1973